Charge dependent relation between the masses of different generations and Neutrino masses *

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Abstract

Despite the enormous achievements, the Standard model of Particle physics can not be consider as complete theory of fundamental interactions. Among other things, it can not describe the gravitational interaction and it depends on 19 parameters. The Standard model includes 12 fermions (matter elementary particles with spin $\frac{1}{2}$) which are divided in three generations, groups with same interactions but different masses. Each generation can be classified into two leptons (with electric charges $Q = -1$, electron-like and $Q = 0$, neutrino) and two quarks (with electric charges $Q = -\frac{1}{3}$, down-type and $Q = \frac{2}{3}$, up-type). However, the understanding of the relationship between generations and ratio of masses of different generations are unknown. Here we show that there exists the simple relation between masses of different generations which depend only on the electric charges for $Q = -1$, $Q = -\frac{1}{3}$ and $Q = \frac{2}{3}$. It is in pretty good agreement with experimental data. Assuming that the same relation valid for $Q = 0$, we are able to calculate neutrino masses. Therefore, our results could pave the way for further investigations beyond Standard model.

1 The basic relation

Although the Standard model successfully explains many experimental results, it can not explain origin of quarks and leptons generations and relation between their masses. In the present article we offer the simple relation which connect the ratio of masses of different generations with their charges.

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It is well known that there are three generations of quarks and leptons in Nature. They appear in equal numbers and with the same interactions in every generation. Only the masses vary, but with significant difference. From neutrino oscillations it is known that neutrinos have masses, but experiments gives only an upper limit.

Table 1: Masses and charges of quarks and leptons (the t quark mass is in GeV, while all other masses are in Mev)

	$Q=-1$		$Q=-\frac{1}{3}$		$Q=\frac{2}{3}$
m_τ	1 776.82 $\pm$ 0.16	m_b	$(\overline{MS})$: 4 180 $\pm$ 30 (1S): 4 660 $\pm$ 30	m_t	173.21 $\pm$ 0.51 $\pm$ 0.71 $(\overline{MS})$: 160^{+5}_{-4} $176.7^{+4.0}_{-3.4}$
m_μ	105.6583715 $\pm$ 0.0000035	m_s	95 $\pm$ 5	m_c	1 275 $\pm$ 25
m_e	0.510998928 $\pm$ 0.000000011	m_d	$4.8\ ^{+0.5}_{-0.3}$	m_u	$2.3\ ^{+0.7}_{-0.5}$

Let us start with the Table 1 for three generation of quark and lepton masses taken from Review of Particle Physics [1]. We propose a simple relation which connect dimensionless mass dependent expression (on the left side) with simple charge dependent expression (on the right side)

$$\frac{M_2^2}{M_1\, M_3} = e^{\frac{5}{2}Q(q-l)}\,. \tag{1.1}$$

Here $M_1 = \{m_e, m_d, m_u\}$, $M_2 = \{m_\mu, m_s, m_c\}$ and $M_3 = \{m_\tau, m_b, m_t\}$ are masses from first, second and third generations, Q is a electric charge while q and l are the quark and lepton numbers. Note that right hand side depend on the square of quantum numbers and consequently does not depend on the particle antiparticle replacement.

Table 2: Theoretical values

$Q=-1, l=1, q=0$	$Q=-\frac{1}{3}, l=0, q=1$	$Q=\frac{2}{3}, l=0, q=1$
$e^{\frac{5}{2}}$ =12,18249	$e^{-\frac{5}{6}}$ =0,434598	$e^{\frac{5}{3}}$ =5,29449

The right hand side of (1.1) is easy to calculate and for different values of charges we obtain Table 2. For the left hand side of (1.1) one should use experimental data. They are well known for charge leptons (electron, muon and tau), they are roughly estimated for quarks and they are not known for neutrinos.

1.1 Lepton sector

In the lepton sector we have

$$\frac{m_\mu^2}{m_e\, m_\tau} = 12,2939\,. \tag{1.2}$$

Because we take into account only electro-magnetic interaction, it is in pretty good agreement with theoretical value.

1.2 Quark sector

The masses in the Table 1 are taken from Ref.[1]. The masses of the first generation $m_u\,, m_d$ and mass of second generation m_s are estimates of so-called "current-quark masses" in a mass-independent subtraction scheme such as $\overline{MS}$ at a scale $\mu \approx 2\ GeV$. The second generation m_c and third generation m_b masses are "running" masses in the $\overline{MS}$ scheme. For the b-quark the $1S$ mass is also quoted. For the third generation m_t quark mass the first value is from direct measurements, the second one is in $\overline{MS}$ scheme from cross-section measurements while the third one is pole from cross-section measurements.

Table 3: Upper bounds, estimated values and lower bounds of $\frac{M_2^2}{M_1\, M_3}$ for quarks

	$Q=-\frac{1}{3}$		$Q=\frac{2}{3}$		
	Max	0,5355		Max	5,4590
$\overline{MS}$		0,4498	Direct measurements		4,0806
	Min	0,3630		Min	2,9859
	Max	0,4800		Max	6,2593
1S		0,4035	$\overline{MS}$		4,4175
	Min	0,3259		Min	3,1566
	Max			Max	5,4177
			Pole from cross-section		4,0000
	Min			Min	2,8823

The upper bound, the estimated value and the lower bound for all cases of quark masses are calculated in the Table 3. We denote Max $\equiv \frac{(maxM_2)^2}{minM_1\ minM_3}$ and Min $\equiv \frac{(minM_2)^2}{maxM_1\ maxM_3}$. In all cases the indeterminacy is not small, but the theoretical values from Table 2 lies within allowed intervals.

2 Neutrino masses

Let us suppose that the proposed expression (1.1) valid for neutrinos, also. Then for $Q = 0$ we have

$$m_{\nu_\mu}^2 = m_{\nu_e} m_{\nu_\tau} \, . \tag{2.3}$$

In the neutrino sector the only known experimental values are data based on the 3-neutrino mixing scheme [1]

$$\Delta m_{21}^2 \equiv m_{\nu_\mu}^2 - m_{\nu_e}^2 = (7.53 \pm 0.18) \times 10^{-5} eV^2 \, , \quad \Delta m_{32}^2 \equiv m_{\nu_\tau}^2 - m_{\nu_\mu}^2 = (2.44 \pm 0.06) \times 10^{-3} eV^2 \, , \tag{2.4}$$

for normal mass hierarchy ($m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau}$) and

$$\Delta m_{32}^2 \equiv m_{\nu_\mu}^2 - m_{\nu_\tau}^2 = (2.52 \pm 0.07) \times 10^{-3} eV^2 \, . \tag{2.5}$$

for inverted mass hierarchy ($m_{\nu_\tau} < m_{\nu_e} < m_{\nu_\mu}$). So, the sign of Δm_{32}^2 is not known experimentally. In our case, from the relation (2.3) we can conclude that it has the same sign as Δm_{21}^2. Therefore, the normal mass hierarchy appears.

Using expression (2.3) we have

$$m_{\nu_e} = \frac{\Delta m_{21}^2}{\sqrt{\Delta m_{32}^2 - \Delta m_{21}^2}} \, , \quad m_{\nu_\mu} = \sqrt{\frac{\Delta m_{21}^2 \, \Delta m_{32}^2}{\Delta m_{32}^2 - \Delta m_{21}^2}} \, , \quad m_{\nu_\tau} = \frac{\Delta m_{32}^2}{\sqrt{\Delta m_{32}^2 - \Delta m_{21}^2}} \, . \tag{2.6}$$

The explicit values are present in Table 4, which is supplement of Table 1 for $Q = 0$.

Table 4: Neutrino masses

	$Q = 0$
m_{ν_τ}	$50.18 \times 10^{-3} eV$
m_{ν_μ}	$8.81 \times 10^{-3} eV$
m_{ν_e}	$1.55 \times 10^{-3} eV$

All results for neutrino masses are in agreement with limitation from tritium decay $m_\nu < 2eV$, [1]. Consequently, if the relation (1.1) is true for neutrinos, we are able to calculate their masses.

3 Conclusion

The central part of the present article is the expression (1.1). It shoes that the dimensionless mass dependent expression of different generations depend only on electrical charge. We are not able to prove this expression theoretically, but we verified it in the cases of

charge leptons and quarks, using experimental data from Ref.[1]. We can conclude that the relation (1.1) is in pretty good agreement with experimental data.

The important prediction follows from the hypothesis that the expression (1.1) valid for uncharged leptons–neutrinos, also. In that case we are able to calculate neutrino masses (see Table 4).

The only parameter in equation (1.1) is $\frac{5}{2}$. We do not inclined to explain the origin of this number, but those who is convinced of the correctness of the string theory can just argue that this is a quotient of number of critical dimensions in superstring theory $D = 10$ and the number of non-compactified dimensions $d = 4$.

Consequently, if our main relation is true, there are 6 instead of 9 independent masses and 16 instead of 19 independent parameters in the Standard model. In addition, we predicted neutrino masses which are not the part of the Standard model. We hope that new relation will help better understood the problem of generations in particle physics. Because the neutrino masses have not yet been measured we expect that future experiments will confirm our prediction.

References

[1] K.A. Olive et.al. (Particle Data Group), Chin. Phys. **C38**, 090001 (2014) (URL: http://pdg.lbl.gov)